Oblique and Yawed Rod Penetration

Gabi Luttwak

RAFAEL, P.O. Box 2250, Haifa, 31021, Israel

The effect of oblique and yawed impact on the penetration of long rods into thick targets is investigated. We review our semi-empirical model [2] which reproduced well the experimental data of Yaziv et al [1] on the dependence of the penetration depth on the yaw angle for normal impact. The feasibility of using a similar approach to treat oblique impact is considered. Using the data of Behner et al [3] and the insight gained from the numerical simulations, we analyze the mechanics of oblique penetration with yaw and consider extending the model to treat the yaw dependence of the penetration in oblique impact. The numerical simulations were carried out, using the second order Godunov processor in the code Autodyn V4.0. The calculations help us follow the flow and to calibrate the analytical model. The comparison with published experimental data also serves to further benchmark the code.

INTRODUCTION

The impact of a high-speed projectile into a target produces high pressures. These, induce a flow of material outward from the point of impact, leading to the deformation of the projectile head and to the formation of a hole in the target. While the rest of the projectile can pass through this hole, the interaction region is limited to the neighborhood of the penetrating head and the penetration depth is proportional to the projectile length, L. Therefore, long rods with a large length to diameter ratio (L/D > 10), are commonly used to defeat thick armor plates. The deepest penetration is obtained for normal impact with the long rods moving along their axis. Oblique and yawed impact can greatly decrease the penetration depth. To predict the performance of these projectiles it is necessary to understand the effects of obliquity and yaw on the penetration. In a previous work [2], we have carried out numerical simulations of yawed rod penetration in normal impact. After analyzing the result of these calculations and examining some published experimental results [1], we have proposed a new, semi-empirical model for the dependence of the penetration depth on the yaw angle. In a recent paper [3], extensive experimental data has been published regarding the combined effect of yaw and 60° oblique impact on penetration. We now carry out numerical simulations.

Experiments have been carried out [1-7]. The results have been analyzed both by analytical models [3-7] and in the recent years more and more by numerical simulations [11]. The normal impact of a rod with yaw into a thick target produces high deformations especially in the rod. Parts of the projectile which were located at separate places along the rod get into contact which each other and with the target as they form the crater. Multi-material Eulerian calculations are best fit to handle such a problem. The available experimental data (Yaziv[1-2]) makes this solution also a good candidate for code validation. In 1994, after the author has participated in the development of the multi-material Euler with strength processor in the 3D code MSC-DYTRAN [8,14], numerical simulations were done in order to study this problem and to serve as another benchmark for code-validation. For the same purposes, similar calculations were now carried out,
using the second order Godunov, multi-material Euler processor of the code AUTODYN-3D[9-10,15]. Making the assumption that the crater volume is independent of the yaw angle and looking at the results of the simulations we formulate a new yaw model which does reproduce well the experimental data of Yaziv[2]. Today it is the role of the numerical simulations to provide quantitative prediction or reproduction of the experimental results. Still, simple analytical models has a definite role, to provide additional physical insight and help determine the most important parameters which influence the results.

**YAW MODEL FOR NORMAL PENETRATION**

Let us first consider the case of normal penetration with yaw into thick targets. The yaw of a projectile is the angle $\alpha$, between the velocity $\mathbf{v}$ and its axis of revolution $\hat{k}$ (this is also called total yaw[3,5]). Thus, $\cos \alpha = \frac{\mathbf{v} \cdot \hat{k}}{\|\mathbf{v}\| \|\hat{k}\|}$. Several analytical models for penetration with yaw have been published [3-8].

Let us briefly review our recently published semi-empirical model on the dependence of the penetration depth on the yaw angle. It is based on one basic assumption. For thick target penetration, all the kinetic energy of the rod is transformed into plastic work to form the hole in the target. Therefore, the crater volume should be independent of the yaw angle. To refine the model, we look at the results of the experiments and simulations and examine the shape of the crater formed in the target at different yaw angles. $L, D$ are the rod length and diameter. Also, $D_h$ and $P(0)$ are respectively the crater diameter and the penetration depth formed in the target by a similar rod penetrating without yaw. Let $k = \frac{D_h}{D}$, be the ratio between the crater and rod diameters. For a penetrating rod, $k \gg 1$. The resulting relation for the penetration depth $P$ as a function of yaw angle $\alpha$ is:

$$P(\alpha) = \frac{\gamma k P(0)(1 + \sin(2\alpha))}{(L/D)\sin(\alpha) + \gamma k \exp\left(-\frac{\alpha}{\alpha_c}\right)} \quad (1)$$

In the above relation, $\bar{\alpha}$ and $\bar{\alpha}_c$ are parameters. For yawed penetration at $\alpha \gg \alpha_c$, the crater size in the yaw direction is $L\sin(\alpha)$. The width of the crater in a perpendicular direction remains of the order of $D_h$. Thus, if the profile of the hole maintains a cylindrical shape, while its volume stays constant, the penetration depth should decrease like $\frac{D_h}{L\sin(\alpha)}$. This $1/\sin \alpha$ dependence is similar to Bless [7]. However the crater profile at $45^0$ has a triangular shape. This is seen both in the experiments and the simulations. To take this into account, we have added to the numerator of (1) a shape factor: $1 + \sin(2\alpha) = \begin{cases} 1, & \alpha = 0,90^0 \\ 2, & \alpha = 45^0 \end{cases}$. If we require, $\frac{D_h}{L\sin(\alpha)} < 1$, we will obtain for the critical angle:

$$\alpha_c = \sin^{-1}\left(\frac{D_h}{L}\right) = \sin^{-1}\left(\frac{k D}{L}\right) \quad (2)$$

For small yaws, the rod will penetrate through the crater formed by the penetrating head without further interacting with the walls behind it. The effect of yaw increases dramatically at angles larger than $\alpha_c$. The
exponential decaying term \( \exp \left( -\frac{\alpha}{\alpha_C} \right) \) in (1) makes the penetration curve smoothly move toward the \( \frac{D_h}{L \sin(\alpha)} \) dependence, while reproducing \( P(0) \) for \( \alpha = 0 \). The parameter \( \bar{a} \) can be evaluated by looking at the \( \alpha = 90^\circ \) penetration data:

\[
\gamma = \frac{L}{D_h} \frac{P(90^\circ)}{P(0)} = \frac{1}{kD} \frac{L}{D} \frac{P(90^\circ)}{P(0)}
\]

Inserting the experimental data of Bjerke for \( D_h \) in normal penetration and the \( P(0) \) and \( P(90^\circ) \) data of Yaziv yields \( \gamma \approx \frac{2}{3} \).

**TABLE I. PENETRATION DEPTH (MM) AS A FUNCTION OF YAW ANGLE**

<table>
<thead>
<tr>
<th>Yaw</th>
<th>0°</th>
<th>4°</th>
<th>7.5°</th>
<th>19°</th>
<th>31°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60</td>
<td>57</td>
<td>52</td>
<td>40.5</td>
<td>30</td>
<td>8</td>
</tr>
</tbody>
</table>

Extracting out \( \bar{a}k \) from (3), we get instead of (1):

\[
P(\alpha) = \frac{P(0)P(90^\circ)(1 + \sin(2\alpha))}{P(0)\sin(\alpha) + P(90^\circ)\exp \left( -\frac{\alpha}{\alpha_C} \right)}
\]

This form, (4), can be compared to the empirical relations (5,6) from ISL:

\[
P(\alpha) = (P(0) - P(90^\circ))\exp(-b\alpha^{1.4}) + P(90^\circ)
\]

or from EMI:

\[
P(\alpha) = (P(0) - P^*) \left( \frac{1}{c_1\alpha^{2.0} + 1} \right) + P^*
\]

In (5) and (6) the exponents 1.3, 2.0 and \( b, p^* \) and \( c_1 \) are fit parameters. Let note that (4) has no arbitrary parameters, except the critical angle \( \alpha_C \), which can be approximately evaluated using (2). From (2) we get for the critical angle \( \alpha_C = 12.12 \). Using this value in (1) or in (4) we get a reasonable fitting for the penetration data of Yaziv [1,4]. Choosing \( \alpha_C = 11.2 \) gives the best fit. In Table I and in Fig. 1 we see an excellent agreement between our model and the experimental data for all angles.

Using (1) has more predictive power than (4) as it shows the L/D dependence of the relation. Moreover, the hole diameter \( D_h \), and the ratio \( k \) can be estimated using the hydrodynamic theory of penetration [2]:

\[
k = \frac{D_h}{D} \approx \frac{v}{\left( 1 + \sqrt{\frac{\rho_T}{\rho_p}} \right) \left( \frac{2Y_T}{\rho_T} \right)}
\]

For a high speed rod penetrating into an infinite target, the hydrodynamic theory yields the well known relation. \( P(0) = L\sqrt{\frac{\rho_T}{\rho_p}} \). Taking into account the secondary penetration at the rod tail, leads to
\[ P(0) = L \sqrt{\frac{\rho_T}{\rho_p}} + D_h \]. The rod can sustain stresses up to its yield strength. Taking into account the deceleration of the rod due to the impact stress, leads at lower impact speeds, \( v < \sqrt{\frac{Y_p}{\rho_p}} \), to a reduced penetration depth which can be approximated by:

\[
P(0) = L \sqrt{\frac{\rho_T}{\rho_p}} \left( 1 - \sqrt{1 - \frac{v^2}{\left(\frac{Y_p}{\rho_p}\right) \left(1 + \sqrt{\frac{\rho_p}{\rho_T}}\right)}} \right)
\]  \hspace{1cm} (8)

The penetration, \( P(0) \), increases with the velocity until it reaches the value \( L \sqrt{\frac{\rho_p}{\rho_T}} \). Combining (1), (7) with (8) the dependence of the penetration on yaw, impact speed, projectile and target densities and target yield strength can be assessed.

![Figure 1](image-url)

**Figure 1.** Penetration for 1.4mm/μs normal impact with yaw of a L=80,D=8mm,Tungsten rod on a RHA steel target

**YAW MODEL FOR OBlique PENETRATION**

Encouraged by the above results, let us proceed with the same approach to oblique penetration. First, we consider oblique penetration without yaw. The obliquity is the angle \( \theta \), between \( \hat{v} \) and the normal \( \hat{n} \) to the target, with \( \cos \theta = |\hat{v} \cdot \hat{n}| \). Oblique impact without yaw has a plane of symmetry. It is the plane defined by \( (\hat{n}, \hat{k}) \). In high speed penetration, the gravity can be safely neglected. Still, it is convenient to define the plane \( (\hat{n}, \hat{k}) \) as vertical with the rod upper side pointed by \( \hat{n} - (\hat{n} \cdot \hat{k})\hat{k} \). For thick target penetration, we again assume that all the rod kinetic energy is used up to form the hole in the target. Thus, the crater volume should be independent of the obliquity angle, \( \theta \). The crater diameter inside the target will be the same as for
normal penetration. For an infinite target, the penetration depth along the rod trajectory should remain, $P(\alpha = 0, \theta) = P(\alpha = 0, \theta = 0) = P(0)$, independent of the obliquity. The penetration perpendicularly to the target face will of course decrease to $P(0) \cos(\theta)$. Looking at the results of the simulations for $\theta = 60^0$, Fig.3-5, we see that the crater is asymmetric. While first penetrating to a depth of about $D_h \tan(\theta)$, on the rod upper side the crater approaches the target’s free surface. This can make it easier to drill the hole into the target, but can make the crater bend toward the free surface. The interaction with the target material at the rod lower size will occur over a larger region. This region will be even larger at lower impact speeds, when the crater size and its rate of growth are slower. Depending on the rod yield stress, these forces at the rod lower size can produce a moment of rotation which tends to turn the rod outward from the target. As the obliquity angle increases over a critical angle $\theta_c$, the rod will turn and bounce back like a ricochet. As we approach $\theta_c$, the penetration depth into the target will quickly decrease. For finite width targets, the rod entrance and target back face regions have a non-negligible effect on the penetration depth. At a depth of $D_h \tan(\theta)$ from the back of the target, the lower part of the crater will approach the target’s back face. We will further investigate these in the future.

Oblique and yawed impact can occur together. Relative to the plane $\hat{n}, \hat{k}$, the yaw angle $\hat{a}$ can be decomposed into the in-plane $a_1$, and the off-plane $a_2$. Considering the plane $\hat{n}, \hat{k}$ as vertical, we call $a_1$ and $a_2$ respectively pitch and horizontal yaw in analogy with the role of these in aeronautics and exterior ballistics. Let $\hat{i} = (\hat{n} \times \hat{k})/|\hat{n} \times \hat{k}|$ be the unit vector normal to this plane. Then, $\hat{v}_i = (\hat{v} \cdot \hat{i}) \hat{i}$ and $\hat{v}_p = \hat{v} - \hat{v}_i$ are respectively the off-plane and in-plane components of the velocity. Let $\hat{v}_p = \hat{v}_p/|\hat{v}_p|$; $\hat{v}_i = \hat{v}_i/|\hat{v}_i|$ be unit vectors along $\hat{v}_p$ and $\hat{v}_i$. Thus, $[\alpha_1] = \cos(\hat{v}_p \cdot \hat{k})$ and $\alpha_2 = \cos(\hat{v}_i \cdot \hat{k})$. From the geometry, $\tan^2(\alpha) = \tan^2(\alpha_1) + \tan^2(\alpha_2)$. The pitch $a_1$ is defined as positive if $|\hat{v}_p \cdot \hat{n}| > |\hat{k} \cdot \hat{n}|$. This decomposition of the total yaw is motivated by the different sensitivity of the penetration on positive pitch, on negative pitch and on horizontal yaw. By invariance under the choice of the frame of reference, combined oblique and yawed impact is equivalent to the oblique impact without yaw with the speed of $\left(\hat{v} \cdot \hat{k}\right)\hat{k}$ into the target moving at a speed of $-\left(\hat{v} - (\hat{v} \cdot \hat{k})\hat{k}\right)$.

Let us consider the effect of yaw on oblique penetration. The crater size in the direction of yaw should increase from $D_h$ to about $L\sin(\alpha)/\cos(\theta)$. While, the crater will form along the rod initial trajectory, its volume will depend only on the projection of the base area normally to this direction, which will increase like $L\sin(\alpha)$ with no dependence on the obliquity angle. Thus we would expect equations (1) and (4) to also hold for oblique penetration with yaw, but perhaps with a different critical yaw. The critical yaw angle $\alpha_c$ will depend both on $a_1$ and $a_2$. For positive pitch, $a_1 > 0$, the rod is deflected upward, away from the target increasing its effective obliquity. At larger obliquities and pitch as seen both in the simulations (Figs. 4-5) and in the data of Behner et al. for the case of $\theta = 60^0$, this makes the rod to turn and the crater bend upward. Thus the penetration depth along the projectile line of sight will decrease quickly. Looking at the experimental data of Behner et al, we can see that the

Let us consider the effect of yaw on oblique penetration. For small obliquities ($\theta << \theta_c$), instead of $D_h$ the crater size in the direction of yaw should increase. If the tail of the rod would move forward with yaw $\hat{a}$, it would hit the target at a distance $L\sin(\hat{a})/\cos(\theta + \hat{a})$ from the rod head. This relation will hold both for positive and negative pitch. For horizontal yaw, this distance would be $L\sin(\hat{a})/\cos(\hat{a})$. While, the crater will form along the rod initial trajectory, its volume will depend only on the projection of the base area normally to this direction, which will increase for a pitch $\hat{a}$, like $L\sin(\hat{a})\cos(\theta)/\cos(\theta + \hat{a})$. Thus we would get instead of (1):

\[ \text{Penetration} = L \sin(\hat{a})/\cos(\theta + \hat{a}) \]
\[ P(\alpha) = \frac{\gamma k P(0)(1 + \sin(2\alpha_i))}{\left(\frac{L}{D}\right)\sin(\alpha_i)\cos(\theta) + \gamma k \exp\left(-\frac{\alpha_i}{\alpha_c}\right)} \]  

(9)

The additional term will make, as expected, the penetration with positive pitch decreases quicker than for negative pitch. For horizontal yaw we would expect equations (1) and (4) to hold for oblique penetration with yaw, but perhaps with a different critical yaw. The critical yaw angle \( \hat{\alpha}_c \) will be smaller for positive pitch than for negative pitch or horizontal yaw. For positive pitch, \( \hat{\alpha}_i > 0 \), the rod is deflected upward, away from the target increasing its effective obliquity and decreasing its penetration. At larger obliquities and pitch as seen both in the simulations (Figs. 4-5) and in the data of Behner et al. for the case of \( \theta = 60^\circ \), this makes the rod to turn and the crater bend upward. As the crater approaches the free surface, the material in the target there may fail. This leads to the increase in the crater volume as seen in the experimental data of Behner. This material failure also contributes to the upward deflection of the rod. At this stage, the penetration depth along the projectile line of sight will quickly decrease, while (9) should predict the crater length which is larger.

The additional term will make, as expected, the penetration with positive pitch decreases quicker than for negative pitch. For horizontal yaw we would expect equations (1) and (4) to hold for oblique penetration with yaw, but perhaps with a different critical yaw. The critical yaw angle \( \hat{\alpha}_c \) will be smaller for positive pitch than for negative pitch or horizontal yaw. For positive pitch, \( \hat{\alpha}_i > 0 \), the rod is deflected upward, away from the target increasing its effective obliquity and decreasing its penetration. At larger obliquities and pitch as seen both in the simulations (Figs. 4-5) and in the data of Behner et al. for the case of \( \theta = 60^\circ \), this makes the rod to turn and the crater bend upward. As the crater approaches the free surface, the material in the target there may fail. This leads to the increase in the crater volume as seen in the experimental data of Behner. This material failure also contributes to the upward deflection of the rod. At this stage, the penetration depth along the projectile line of sight will quickly decrease, while (9) should predict the crater length which is larger. Looking at the experimental data of Behner et al, we can see that the penetration yaw curves look similar to the corresponding

THE NUMERICAL SIMULATIONS

In a previous work [2], we have carried out numerical simulations of normal impact with yaw. We now investigate the effect of yaw on penetration for 60° oblique impacts. Following the experimental data of Behner et al., we consider Tungsten rods moving with 1600m/sec into thick steel targets. The rod are 80mm long and has \( L/D = 20 \). is available for this problem. By If there is no horizontal yaw \( \hat{\alpha}_i = 0 \), then the plane \( \left(\hat{n}, \hat{k}\right) \) is a symmetry plane for the problem. Thus, The shock equation of state was used in the AUTODYN simulation. The strength calculations utilized the Steinberg [12] model which allows for strain hardening and take into account the dependence of the yield strength on pressure and temperature. No failure model was defined in the simulations. The default parameters from AUTODYN’s library were taken for Tungsten and for SS-21-6-9 steel. A fixed, 1mm zone size was use in all the calculations.

The results of the three dimensional AUTODYN simulations are shown in Figs.4,5. The evolution of the shape of the crater as seen on the symmetry plane (Fig.5) is quite similar to the result of the 2D plane strain simulations (Figs. 2,3). The final shape is also similar to the experimental data (Fig.1). The depth of penetration is 22mm at 50\( \mu \)s and 22.2mm at 60\( \mu \)s. The value is smaller that the 27mm experimental value in Fig. 1. The differences might be caused also by the material parameters used. This value is smaller than the 32mm obtained with the 2D plate impact simulations. In Fig.4 we see the cross section of the crater in several planes perpendicular to and cutting the symmetry plane. Of course, this kind of information can be obtained only from the 3D simulation. The shape and the sizes of the crater compare reasonable well to the experimental data in Fig.1. We do not show here the older, DYTRAN simulation. In 1995, we had to make a variable mesh, to keep the problem in the available workstation memory. Therefore, we limited the 1mm resolution to the actual interaction region. Early time result were satisfactory, but after 40\( \mu \)s the penetrating head got to a coarser mesh region. Due to the larger memories of today's computers and taking advantage of the virtual memory scheme [9-10] in AUTODYN V4, there were no similar problems in the current calculation.

CONCLUSIONS

The numerical simulations carried out in either DYTRAN or AUTODYN reproduce the essential features of yawed rod penetration. Fine tuning the material model will improve the quantitative comparisons. The solution of two-dimensional plane strain calculations provide quickly many of the essential features of the 3D problem. At higher impact speeds, the larger hole diameter of plate impact makes it less sensitive to the effect of yaw relative to the case of rod impact.
We formulated a new model for yaw impact. It is based on the reasonable assumption of the hole volume being independent of the yaw. When taking into account the typical shape of the hole, the model could be fitted to reproduce excellently well the experimental data. The real test of this model will be to predict other configurations with different materials, impact speed or (L/D). We consider using similar reasoning to model oblique rod penetration.

REFERENCES


To analyze this process, several experiments have been carried out [1-7]. The results have been analyzed either by analytical models [3-7] or, more and more by numerical simulations [11].