VIP (Vector Image Polygon) multi-dimensional slope limiters for scalar variables

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Abstract

A recently formulated \textit{frame-invariant} monotonicity criterion and slope limiter for vectors was applied to the Staggered Mesh Godunov-SMG/Q scheme for Lagrangian and ALE hydrodynamics. The VIP (vector-space polygon or polyhedron) was shown to be a natural extension of monotonicity constraints from scalar to vector variables. Taking notice of the fact that gradients of scalars are vectors, we now seek to use this new concept to devise better, and perhaps \textit{truly multidimensional}, slope limiters for scalar variables. The proposed scheme constitutes a generalization of a 1D monotonic-averaging limiter for scalars, to a VIP type limiter for (vector) gradients of scalars in 2D or 3D. Test cases computed by the SMG/ALE scheme, using the new VIP limiter for gradients, are presented. As we can see from them, the new method, while being robust in strong shock computations, can better preserve gradients of density under advection.

\textit{Keywords:} VIP limiter, Lagrange hydrodynamics, ALE, SMG scheme

1. Introduction

In finite volume schemes the conservation laws are integrated over some control volume. For example, the equation of mass balance for a zone $c$ moving with the mesh velocity $\vec{u}_g$ is

$$\frac{d}{dt} \int_c \rho \, dV = \int_{\partial c} \rho (\vec{u} - \vec{u}_g) \cdot d\vec{s},$$

(1)
where $\partial c$ is the boundary of zone $c$ and $\rho$, $\vec{u}$ are the material density and the velocity, which in a high order scheme are taken to vary in zone $c$. To compute the mass flux across $\partial c$, the face-centered density values are required. In a second order scheme these are usually obtained from the upstream values of zone-centered variables, extrapolated to points $\vec{r}_x$ of the zone boundary from the zone center $\vec{r}_c$ by the zone-centered gradient:

$$\rho_x = \rho_c + \left[\nabla \rho\right]_c \cdot (\vec{r}_x - \vec{r}_c).$$

In solutions to the fluid dynamical equations discontinuities (shock or contact) may initially be present, or can arise during the flow. At such discontinuities gradient values are unbounded. Hence, using unlimited values of gradients in Eq. 2 results in a computational scheme where captured discontinuities usually involve non-physical fluctuations. Therefore, the gradients in Eq. 2 must be limited to maintain monotonicity of the numerical solution near captured discontinuities.

Slope limiters were first formulated in one dimension, and were readily extended to multi-dimensional Cartesian mesh by operator splitting [1, 2]. Many of the issues arising in the formulation of a multidimensional slope limiter for non-Cartesian and unstructured meshes have been already investigated (see [3, 4, 5, 6, 7]). For vector or tensor variables the common approach was to consider the limiting of each component separately. Such limiting is not invariant under rotational transformation of vector or tensor variables, and hence violates symmetries present in the problem (see [8, 9, 10]). A VIP limiter, being invariant under rotational transformations, generally improves symmetry preservation. This symmetry breaking caused by limiting each vector component separately has been noticed also by Loubere et al. (see [11, 12]). Their solution is to choose vector components aligned with flow-related directions. That procedure is simpler to implement than the VIP and will also prevent symmetry breaking. However the choice of those flow-aligned vector components is to some extent arbitrary. By contrast, we have shown that VIP is the natural extension of a one dimensional range to a multidimensional set-up.

Turning to scalar variables, and considering first the one-dimensional case, there are two distinct approaches to define a slope (gradient) limiter, which we denote here as $\mathcal{MA}$ (monotonic averaging) and $\mathcal{ME}$ (monotonic extrapolation). Following van Leer [13], consider the $\mathcal{MA}$ approach for the three consecutive zones $j-1, j, j+1$ in a uniform grid with mesh size $\Delta x$, and let
\[
a = \frac{\rho_j - \rho_{j-1}}{\Delta x}, \quad b = \frac{\rho_{j+1} - \rho_j}{\Delta x},
\]
be, respectively, the left and right slopes of the zone-centered density \( \rho_j \). Then a monotonic average slope \( \text{ave}(a,b) \) for zone \( j \) is given by
\[
\text{ave}(a,b) = \minmod \left( \frac{1}{2} (a + b), 2a, 2b \right).
\]  
(3)

Note that the definition of \( \minmod(\cdot, \cdot, \cdot) \) includes a zero condition (denoted hereafter \( ZC \)): when the arguments differ in sign, the \( \minmod \) value vanishes. We shall reconsider this point in the multidimensional case, where the \( ZC \) assumes a more intricate formulation.

The alternate \( M\bar{E} \) formulation consists in requiring that the reconstructed density profile in a zone \( c \) (i.e., the linear slope in the 1D case) remains monotonic with respect to the zone-centered values in all neighboring zones. Specifically, face-extrapolated (or node-extrapolated) densities obtained from Eq. 2 by using the limited gradient in \( c \) must stay in the range spanned by the zone-centered densities in \( c \) and all its face-neighbor or node-neighbor zones \( \nu \). Denoting by \( \bar{r}_x \) the face-extrapolated or node-extrapolated point, the limiter is
\[
\min_{\nu} (\rho_\nu) \leq \rho_c + \left[ \tilde{\nabla} \rho \right]_{c}^{\lim} \cdot (\bar{r}_x - \bar{r}_c) \leq \max_{\nu} (\rho_\nu). \]  
(4)

vanLeer([13]) defines a range of limiters. The factor of two in the “double minmod” Eq. 3 makes it less dissipative than the minmod limiter \( \text{ave}(a,b) = \minmod(0.5(a + b), a, b) \), and with this factor it is readily shown that in a 1D equally-spaced grid the two approaches (\( MA \) and \( M\bar{E} \)) are equivalent. This ensures that Eq. 3 will preserve the positivity of the density reconstruction in the zone.

Turning to the multidimensional case, most limiters are based on the \( M\bar{E} \) formulation Eq. 4 (e.g., [3],[4]). The algorithm consists in seeking the largest “mitigating factor” \( \alpha \) (for a zone \( c \)) such that the limited gradient complying with Eq. 4 is given by the following \( \alpha \)-contraction of the unlimited gradient:
\[
\left[ \tilde{\nabla} \rho \right]_{c}^{\lim} = \alpha \left[ \tilde{\nabla} \rho \right]_{c}, \quad 0 \leq \alpha \leq 1, \]  
(5)
where the unlimited gradient in zone \( c \) can be obtained for example from the surface integral of the face density (average of the zone densities on either side).

Scalar limiting, where \( \alpha \) is defined by requiring Eq. 4 at all nodes of a zone, is simple to implement and is our default scheme. However in many cases
it might be excessively dissipative (see test case 3.1 below and also Berger et al.[7]). We contend that in case of a shock (or other discontinuity), the limiting should be done preferentially in the front-normal direction, so that the limited gradient would turn away from that direction. Thus, the limiting of a vector variable should be generalized to allow for a rotation, in addition to the contraction factor $\alpha$ previously defined. In other words, we should seek a limited gradient closest in some sense to the unlimited one, that also complies with Eq. 4 for all zone $c$ nodes. Rider and Kothe [5] accomplished that by a Least Square Reconstruction, and an alternate Linear Programming formulation was proposed by Berger et al.[7]. The gradient of scalars are vectors. The vector space region allowed by Eq. 4 for the extrapolating gradient to each zone node is in fact a polygon or a polyhedron. A limited gradient is then taken as the point in this allowed region of vector space, “closest” in some sense to the unlimited gradient.

We propose a different approach based on our VIP (vector image polygon or polyhedron). By starting out from the $\mathcal{MA}$, rather than the previously outlined $\mathcal{ME}$ formulation, we expect to avoid some difficulties associated with the $\mathcal{ME}$ approach, such as discussed by Berger et al.[7]. A multidimensional extension of $\mathcal{MA}$ must be based on a corresponding extension to the “allowed range” in 1D. The VIP domain, being frame invariant under rotation of the coordinate system, is a “natural” extension of a 1D range to a vector-space setup. The present study consists in using the VIP approach to define a monotonic averaging ($\mathcal{MA}$) for the gradient of a scalar variable, thereby enabling direct extension of the (1D) limiter Eq. 3 to a multidimensional setup, to obtain a less dissipative scheme than the scalar limiter Eq. 4.

2. Theory

Consider the application of the $\mathcal{MA}$ algorithm given by Eq. 3 in the 1D case, focusing in particular on the “zero condition” ($\mathcal{ZC}$). While the $\mathcal{ZC}$ is already included in Eq. 3 by the $\text{minmod}$ function, a multidimensional extension of this algorithm cannot be based on the (scalar) $\text{minmod}$ function, and has to be formulated explicitly. By contrast, under the previously outlined $\mathcal{ME}$ formulation a $\mathcal{ZC}$ arises “naturally”, as only $\alpha=0$ can satisfy Eq. 4 when $\rho_c$ is extremal relative to $\rho_\nu$ at all neighbor zones $\nu$. Thus in a “scalar” limiting based on $\mathcal{ME}$, a $\mathcal{ZC}$ is imposed if there is a monotonicity violation in any direction. This can be overly dissipative as small fluctuations in the tangential direction would unnecessarily zero the zone gradient.
in an otherwise smooth region. On the other hand applying a \( \mathcal{ZC} \) only if there is a slope sign change in all directions (that is only if the origin of the vector space is inside the VIP) can produce an unstable scheme as it would not perceive a planar discontinuity with small perturbations in the transversal direction. We decided to look for a slope sign change in the unlimited gradient direction. This direction is a property of the flow field and is independent of the mesh orientation. Briefly stated, our \( \mathcal{MA} \) algorithm is based on a two-stage formulation as follows. The first stage consists in identifying a pattern requiring \( \mathcal{ZC} \), in which case the limited gradient is set to zero. In non-\( \mathcal{ZC} \) cases we proceed to the second stage – the construction of the VIP and subsequent determination of the limited gradient.

Assuming that zone \( c \) might be in the midst of a captured discontinuity, let us define the unit vector \( \hat{n} \) parallel to the unlimited gradient in zone \( c \) as the normal to the front of that wave. Then, let \( \vec{a}_\nu \) be the unlimited gradients in all neighbor zones \( \nu \), and let the projections of these vectors on \( \hat{n} \) be given by \( s_\nu = \hat{n} \cdot \vec{a}_\nu \). Then a \( \mathcal{ZC} \) situation is declared whenever \( s_{\text{min}} \leq 0 \leq s_{\text{max}} \), where by \( s_{\text{min}}, s_{\text{max}} \) we denote the minimum, maximum values of the sequence \( s_\nu \) for all neighbor zones \( \nu \).

In the second stage, dealing solely with non-\( \mathcal{ZC} \) cases, we proceed to construct the VIP. Thus, the scalar expression \( \text{minmod}(0.5(a + b), 2a, 2b) \) should be replaced by the VIP (convex hull in vector space) generated by twice the neighbor zone slope vectors and set the limited gradient in zone \( c \) as the nearest point on the VIP. An outline of the VIP construction is provided in the sequel.

2.1. The VIP-The Vector Image Polygon/Polyhedron of a set of vectors

The VIP was originally proposed [8, 9] as an extension of a 1D range to a vector-space setup. A given sequence of scalar variables \( s_\nu \) is always contained in the range \([\text{min}(s_\nu), \text{max}(s_\nu)]\). By analogy, the corresponding range for a sequence of vectors \( \vec{a}_\nu \) is the vector-space polygon (or polyhedron) constructed as the convex hull of the set of points given by \( \vec{a}_\nu \). For the density-gradient vectors considered here, in analogy to Eq. 3 of the scalar case, the vector sequence is \( 2\vec{a}_\nu \), where \( \vec{a}_\nu \) is the unlimited density gradient in each neighbor zone \( \nu \) to zone \( c \). The VIP concept is illustrated in Fig. 1, and the reader is referred to [8] for further details.
2.2. The Lagrangian phase of the SMG/Q scheme

The Staggered Mesh Godunov SMG/Q scheme [14, 15, 8, 9, 10], uses a staggered mesh (Fig. 2) and is formulated in terms of internal energy. The node-centered velocities are assumed to produce a piecewise linear distribution with possible discontinuities at the in-cell corner zone interfaces (e.g., A in Fig. 2) which separate each pair of edge-neighbor vertices, such as \((i, i+1)\) in zone \(k\). Zone centered velocity gradients \((\nabla \vec{u})_k\) are calculated from the zone vertices velocities. These gradients, are limited using the new VIP frame invariant limiter to preserve a monotonic velocity distribution. The limited
slopes are then used to compute the velocity jump at A which serves as data for a simplified “Impact Riemann Problem” (IRP) there. The IRP is solved in the normal to shock direction which we assumed to be along the velocity difference between the pair of edge neighbors vertices $\Delta \vec{v}_{i,i+1}$. The resulting pressure $p^*$ acts on the corner zone faces. Integrating the contributions from all corner zone faces surrounding a vertex $i$, would directly give its time-advanced vertex velocity. We split this process into two stages: setting $p^*=p_k + (p^* - p_k)$, the contribution of zone pressures $p_k$ is first integrated around vertex $i$. Then the additional term $Q=p^* - p_k$ is treated as a uniaxial pseudo-viscosity, exerting a force along the normal to shock direction taken along the edge neighbors velocity difference $\Delta \vec{u}_{i,i+1}$. The work done by these forces is used to update the zone internal energy like in “compatible hydrodynamics” [16]. The Q-forces thus generated by the SMG scheme tend to “naturally” attenuate hourglass instabilities [14].

2.3. The ALE advection phase

For zone centered variables we compute the fluxes through the zone faces. The volume flux at each zone face is defined as the volume swept by that face when its nodes are moving, respectively, with the local fluid velocity $\vec{u}$ and mesh velocity $\vec{u}_g$. The densities at the fluxed volume center are evaluated by Eq. 2 from the upstream zone side using the limited gradient. Previously we used scalar limiting as in Eq. 5. In the current test calculations we compare these with the new VIP (gradient) limiter for scalar variables. Face based fluxes do not directly account for the “diagonal flow” between neighbor zones that do not share a common face. This can induce a second order error. In a Cartesian mesh, updating the mesh immediately after alternate directional sweeps can prevent this error. However in a curvilinear mesh such a procedure would violate the Debar criterion. That is, if the projected face areas of a left/right pair (i.e., projection with respect to a flux-aligned unit vector) are different, then in a region with constant density and fluid velocity, a split-divergence calculated from that face pair would produce perturbations in the otherwise uniform flow. In the partially split volume integration scheme [17] only the changes due to the partial fluxes that obey the Debar criterion are immediately updated. The remaining flux-divergence terms are saved and added to all zones at the end of the ALE advection phase. A similar “balancing process”, based on a split-consistency concept, is described in [18]. Margolin and Shashkov [19] achieve second order accuracy by an additional advection step using a monotonicity preserving error compensa-
tion algorithm. We emphasize that slope limiting for zone centered scalar variables is required solely during the ALE advection phase.

The momentum advection is carried out in the staggered mesh. That is the momentum fluxes are computed at the (in-zone) corner zone faces (see Fig. 2). To maintain consistency these volume fluxes are not explicitly computed but the mass volume flux at the zone faces is split into the corresponding corner zone faces and the VIP limiter for vectors is used to get the limited velocity gradient used to obtain the value of the fluxed velocity (see [9] for details).

3. Test Calculations

Margolin and Shashkov (see [19]) present a set of test problems for the advection of a scalar field over a moving ALE mesh. These test cases are not affected by the Lagrangian phase or the momentum advection phase in the code and thus they constitute a direct and sensitive test of the new scheme presented here\(^1\). In this article we specifically consider the case of advection of a density peak over a cyclically moving mesh. Next we test the performance of the scheme for simulations of flows in the presence of strong shocks. Saltzman [20], Sedov [21] and Noh [22] problems test the propagation of essentially 1D shocks over non-aligned or distorted meshes. Our new “Noh corner” problem (a 2D/3D collision Riemann problem) consists of sector-wise converging flows. As VIP for scalars is used only in the advection phase we adapt some of the tests to ALE computation. Since the frame-invariant VIP limiter for vectors is superior to traditional component-wise limiting we use it in all the calculations. Thus in the following tests we compare the use of the existing scheme of “scalar limiting” for scalars (hereafter denoted scheme $B$), to the use of the VIP limiter for both vectors and the (vector) gradients of scalar variables (hereafter denoted as scheme $A$). In scheme $B$ scalar limiting is used for the density and specific internal energy advection. The SMG/Q Lagrangian phase and the momentum advection are carried out with the VIP limiter for vectors in both schemes.

3.1. The Margolin-Shashkov density peak advection test

An initially uniform $100 \times 100 \times 1$ mesh spans a $[1 \times 1 \times 0.01]$ box filled with a cold ideal gas. Its density distribution is defined by a peak function

\(^1\)We thank the reviewer who brought them to our attention.
as: \[ \rho(r) = \max[0.001, 4(0.25 - r)] \] with \( r = \sqrt{(x - 0.5)^2 + (y - 0.5)^2} \). The mesh is cyclically moving with \( x(\xi, t) = (1 - \alpha(t))\xi + \alpha(t)\xi^2 \) and \( y(\eta, t) = (1 - \alpha(t))\eta + \alpha(t)\eta^2 \) with \( \alpha(t) = 0.5 \sin(4\pi t) \), where \( 0 \leq \xi \leq 1, 0 \leq \eta \leq 1 \) are normalized mesh indices \([i/i_{\text{max}}, j/j_{\text{max}}]\) and \([i = 0, 1, 2, \ldots, i_{\text{max}}], [j = 0, 1, 2, \ldots, j_{\text{max}}]\).\(^2\) We first ran this test with our regular one step volume integration. These results can be seen in Fig. 3. In this and all the following iso-density plots we use the ten color bars shown in Fig. 3, with the density varying from \( \rho_1 \) to \( \rho_{10} \) with the respective values given in the figure captions. While the density near the peak is higher with the new scheme \( \mathcal{A} \), the shape of the peak becomes distorted. This calculation tended to become unstable and we had to check the ZC along the flux direction to stabilize it. Did the better resolution in scheme \( \mathcal{A} \) make it more sensitive to the diagonal fluxing errors? Indeed, on Fig. 4 we can see the same results with the partially split volume integration. Now we clearly see that scheme \( \mathcal{A} \) is superior to scheme \( \mathcal{B} \). The density peak is less eroded and also the peak shape remains symmetric.

![Figure 3: Density peak advection. Isodensity (\( \rho_1 = 0, \rho_{10} = 0.9 \)) at \( T=1 \). Unsplit volume integration. (a) Calculated by scheme \( \mathcal{A} \). (b) Calculated by scheme \( \mathcal{B} \).](image)

\(^2\)We take here \( \xi^2 \) instead of \( \xi^3 \) in [19] producing symmetric mesh stretching in \( x \) and \( y \).
Figure 4: Density peak advection. Isodensity ($\rho = 0, \rho_10 = 0.9$) at $T=1$. Directionally split advection.

(a) Calculated by scheme $A$. (b) Calculated by scheme $B$.

3.2. The standing Shock Saltzman test

In the original test [20] a piston moves with a velocity of $u=10$ into a cold $\gamma=5/3$ ideal gas, generating a planar shock that propagates at speed $U_s=13.33334$. This 1D problem is solved on a Lagrangian highly skewed 2D $100 \times 10$ mesh. It is a hard test for schemes prone to hourglass instabilities. However the new VIP limiter for scalars is applied only in the ALE advection phase. Since ALE mesh smoothing would regularize the (skewed) mesh before arrival of the shock wave, we modified Saltzman test to a fixed (skewed) mesh setup, where the shock is stationary, with suitable inflow/outflow boundary conditions as shown in Fig. 5. The initial data for a stationary shock positioned at the midpoint is

$$[p, \rho, u]_L = [133.333, 4, -3.333] ; \quad [p, \rho, u]_R = [0.00667, 1, -13.333].$$

Figure 5: The mesh for the standing shock Saltzman test
Fig. 6 shows the pressure and the density profiles along the bottom and top planes for both calculations. Fig. 7 is isodensity plots for the two cases.

![Figure 6: Standing shock Saltzman test.](image)

(a) Pressure profile at $T=0.4\mu s$
(b) Density profile at $T=0.4\mu s$

Figure 6: Standing shock Saltzman test. $p(x), \rho(x)$ along bottom and top planes

![Figure 7: Standing Shock Saltzman test. Isodensity plot ($\rho_1=0.4, \rho_{10}=4.4$) at $T=0.4\mu s$.](image)

(a) Calculation by scheme $A$.  (b) Calculation by scheme $B$.

3.3. Cylindrical (2D) Noh Test

Cold ideal gas ($\gamma = 5/3, e=0$) collapses radially with $[p, \rho, u_r]=[0, 1, -1]$, producing a strong outgoing shock propagating at speed $U_s=1/3$.

The exact solution is:

$$\rho(r) = \begin{cases} 
16 & \text{if } 0 \leq r < t/3 \\
(1 + t/r) & \text{if } t/3 < r < t 
\end{cases}$$  

A 100 × 100 domain is meshed with 50 × 50 zones. The ALE grid smoothing is kept small to preserve the resolution at and behind the shock. In Fig. 8 we compare the isodensity plots at $T=60$ for both calculations. In Fig. 9 we
Figure 8: 2D Noh Test isodensity plot ($\rho_1 = 1.0, \rho_{10} = 0.25$) at $T=60$. (a) Calculated by scheme $A$. (b) Calculated by scheme $B$.

Figure 9: 2D Noh Test isodensity plot ($\rho_1 = 8.0, \rho_{10} = 17.0$) at $T=60$ the central portion in detail. (a) Calculated by scheme $A$. (b) Calculated by scheme $B$.

show the details of the isodensity plot for the central portion of the mesh. The results in both calculations seem similar. The asymmetry seen is due to the Lagrangian phase and in the SMG/Q scheme it is less severe than in some other staggered mesh schemes [23] using an edge defined viscosity. The use of tensor viscosity [24] or cell centered Godunov techniques [25, 26] might reduce this asymmetry.

3.4. “Noh Corner Test”

A ”cold” ($e = 0$) or ”hot” ($e = 0.3$) ideal gas ($\gamma = 5/3$) in 2D space moves toward the origin with a constant diagonal velocity in each quadrant
as shown in Fig. 10. The initial data is \([p, \rho, u, v]=[0, 1, -\sqrt{1/2}, -\sqrt{1/2}]\). Due to the symmetry, only one quadrant has to be considered. We carried out these simulations in a frame in which the gas is initially stationary and the walls are moving. We call this “Noh Corner” test as for the cold gas, it will converge to the Noh problem solution when the plane is divided into a large number of equal sectors. For this “Noh corner” test we do not know the analytic solution, but the solution is self similar \(^3\) and it is two-dimensional, giving rise to planar and Mach shock reflections. Fig. 11 shows isodensity plots with 100×100 mesh, using either scheme \(\mathcal{A}\) or scheme \(\mathcal{B}\). For the “hot” gas corner flow Fig. 12 displays the isodensity plots of both calculations at \(T=70\). For both the ”cold” and ”hot” gas, the results are similar but for the ”hot” gas, the results with scheme \(\mathcal{A}\) tend to have a somewhat better resolution.

\(^3\)a flow is self similar if \(U(x, y, t > 0) = U(x/t, y/t, t = 1)\) for any field variable \(U\)
3.5. 3D Sedov-Taylor blast wave problem

The initial data is \([p, \rho, u_r] = [0, 1, 0], \gamma = 5/3, e = 5027.7\) in a single zone at the origin. The physical space is a 2.5\(^3\) box divided into a uniform 90\(^3\) mesh. The 3D computation is conducted in the 1.25\(^3\) sized \([x \geq 0, y \geq 0, z \geq 0]\) octant. The exact solution has a post-shock density of \(\rho = 4\) and the front radius reaches \(R = 1\) at \(T = 1\). To keep the mesh fine near the shock, the ALE motion was restricted to start at \(\rho \leq 0.25\). The results are shown in Fig. 13.

3.6. 2D Sedov-Taylor blast wave problem

The initial data is again \([p, \rho, u_r] = [0, 1, 0], \gamma = 5/3, e = 0\). To remove any effect of boundary conditions on symmetry planes, we ran the simulations in the full 2D domain: \([-25 \leq x \leq 25, -25 \leq y \leq 25]\). Again the initial data was \(e = 5027.7\) at the 4 central zones. The stronger cylindrical shock reaches \(R = 1\) at \(T \approx 0.24\). In Fig. 14 we see the isodensity plot at \(T = 0.24\). In Fig. 15 we see more in detail the isodensity plot of the central portion. In both cases we compare the new VIP based limiter for scalars calculation to the default scalar limiter case.

4. Conclusions

We have devised a new VIP based limiter for the gradients of scalar variables. This limiter is applied if either the unlimited gradient lies outside
Figure 13: 3D Sedov Taylor blast wave problem. Isodensity plot with $(\rho_1 = 0.04, \rho_{10} = 0.4)$ at $T=1$.

(a) Calculated by scheme $A$. (b) Calculated by scheme $B$.

Figure 14: 2D Sedov-Taylor blast wave problem: isodensity plot with $(\rho_1 = 0.0, \rho_{10} = 3.6)$ at $T=0.24$. 
the VIP (convex hull in vectors space) spanned by *twice* the gradients in the neighbor zone or if $\frac{\partial \rho}{\partial \hat{n}}$ changes sign ($\hat{n}$ is the unit normal to the shock front). Near a shock the second criterion dominates. In this case we do not have to compute the convex hull and we zero the limited gradient. As we demonstrated by the density peak advection, the new limiter is much less dissipative than the (former) “scalar limiter”. We also learn from this test case that being less dissipative, the new limiter is more sensitive to other errors in the advection scheme and specifically it works better with the partially split advection scheme. The following examples test the performance of the new limiter for strong shock driven compression fluid flows. In these cases the results do depend also on the Lagrangian and the momentum advection phases. In all the examples studied the new scheme performed well. It was stable with reasonable results sometimes similar to scheme $B$. The present limiter was applied once to the zone-centered gradient. Thus the same limited gradient is used for the flux computation at all zone faces. Other variants, like a directional limiter should also be considered.

**References**


