A Shock Aligned Cell Centered Godunov Scheme for Eulerian Hydrodynamics

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Cell-centered Godunov schemes

- Classical Eulerian cell-centered Godunov schemes solve 1D Riemann problems (RP) at the computational zone faces.
- These RP solutions provide the face and time step centered values of the variables required for the integration of the conservation laws.
- These RP are solved in the normal to face direction.
The Classical Godunov Scheme

- This introduces some **mesh dependence** in the solution.
- This was already pointed out by Phil Roe [5]
- Numerical effects result like carbuncle phenomenon and even–odd decoupling see Quirk [6]
- Schemes like Colella’s CTU[7] and rotated Riemann solvers (e.g. Levy et al. [8], Leveques et al. [9], Helzel et al. [10], Ren[11]) try to alleviate this problem.
- Also “Residual-distribution schemes”, see the review of van Leer [12], or Remi Abgrall in this symposiun
Rotated Riemann Solvers

- Relatively Complex schemes
- Problems near strong shocks
  - Perhaps caused by non-invariant slope limiters for vectors
- Could we apply some of the advantages of the SMG scheme to Classical Cell Centered Godunov schemes?
The Staggered Mesh Godunov-SMG Scheme

- The Staggered Mesh Godunov-SMG scheme [1-4] solves the impact RP at the staggered zone faces along the normal to the shock direction.
- This direction is approximated to lie along the velocity difference across that face.
- It uses a convex-hull based VIP rotation invariant slope limiter for vectors.
- The SMG scheme better preserves the symmetry near shocks and also controls hourglass instabilities.
- The aim of the present investigation is to apply similar ideas to classical cell centered Eulerian schemes.
Toward a “Shock-Aligned” Cell-Centered Godunov Scheme

- Variables & gradients defined at cell center.
- Face centered values obtained using the limited gradients.
- RP are solved at the face centers in the shock direction.
- Shock direction (if present) assumed to lie along local velocity difference \((\vec{u}_R - \vec{u}_L)\).
- We use a directional slope-limiter.
- Slope limiter for vectors based on the VIP.
- Here we implement this for a Cartesian mesh.
- We do a directionally split calculation.
The RP Solutions

- When $|\vec{u}_R - \vec{u}_L| < k(c_L + c_R)$ with $c_L$, $c_R$ the L,R sound speed and e.g. $k \approx 0.10$ we take $\hat{n}$ normal to the face.

- Otherwise, $\hat{n} = (\vec{u}_R - \vec{u}_L)/|\vec{u}_R - \vec{u}_L|$

- Data for RP obtained at face L,R sides from the cells C=L,R

  $$\rho_{F,C}^{\text{ext}} = \rho_C + \left(\nabla \rho \right)_{\text{lim}} \cdot (\vec{r}_F - \vec{r}_c) \quad ; \quad C = L, R$$

- Shown for density, but is similar for other variables.

- The RP solutions used to time-integrate the conservation laws.
The “directional” limiter

- Extrapolating variable values from cell C to its faces N:

\[
\rho_{N}^{\text{ext}} = \rho_{C} + \left[\nabla \rho\right]_{C}^{\text{lim}} \cdot (\vec{r}_{N} - \vec{r}_{C}); \left[\nabla \rho\right]_{C}^{\text{lim}} = \alpha \left[\nabla \rho\right]_{C}
\]

- Take the largest \(0 \leq \alpha \leq 1\) such that the extrapolated values \(\rho_{N}^{\text{ext}}\) make no monotonicity violations.

- For a directional limiter, in each direction, N=L,R are the cells on left and right and different \(\alpha\) taken in each direction (I,J,K for a Cartesian mesh)
VIP limiter for vectors

- For scalars the monotonicity criterion requires:
  \[ \min(\rho_v) \geq \rho_{c}^{ext} \leq \max(\rho_v) \quad \text{for } \forall \ v(c) \]

- For vectors, like the velocity \( \vec{u} \), it implies:
  \[ \vec{u}_{c}^{ext} \in VIP\{\vec{u}_v\} \]
  i.e. \( \vec{u}_{c}^{ext} \) must lie inside the **convex hull** generated by the velocities in cell \( c \) and its neighbor cells \( v \).
3D Sedov-Taylor blast wave problem

The initial data is: $[\rho_0, \rho, u] = [0, 1, 0]$; $\gamma = 5/3$; $e = 0$ cold ideal gas with a “point source” $e = e_0/V_0$ in a single zone at the origin.

With $e_0 = 0.244186$ and $V_0$ the zone volume, the shock will reach the radius $R = 1$ at $T = 1$.

The physical space is a $25^3$ box divided into a uniform $150^3$ mesh; $e = 59769.53$ at the origin.

The 3D computation conducted in the $x, y, z \geq 0$ octant with the symmetry planes at $x, y, z = 0$.

The exact solution has a post-shock density $\rho = 4$. 
3D Sedov-Taylor test $T=1$, isobars
With the same initial data, the stronger cylindrical shock will reach the radius $R = 1$ at $T = 0.24$.

The physical space is a $25^2$ box divided into a uniform $180^2$ mesh. The 2D computation conducted in the $x \geq 0, y \geq 0$ sector with the symmetry planes at $x = 0, y = 0$.

We also compare with a Lagrangian SMG calculations.
2D Sedov-Taylor test $T=0.24$, isobars
2D Sedov-Taylor test $T=0.24$, iso-density
Iso-density plot for the Lagrangian SMG 2D Sedov test calculation
The Sedov-Taylor Test Problem

- With an Euler mesh:
  - The mesh must be fine enough to resolve the expanding shock. (AMR technique could help)
  - The scheme must not be too dissipative, as then the shock loses some of its strength and advances slower.
  - The symmetry of the problem must be preserved
- The shock-aligned cell-centered Godunov results are promising.
The Sedov-Taylor Test Problem

- In a Lagrangian Calculation:
  - The challenge is to prevent mesh-deformation
  - The Lagrangian mesh follows the material motion and can resolve the shock well

- With the SMG scheme, a Lagrangian calculation can proceed and the shock is better resolved. Also the symmetry is well preserved.
The Shock-aligned Godunov scheme has a potential to improve present cell centered Godunov schemes. The choice of the slope limiter, and specifically the use of a rotation invariant VIP limiter for vectors is important too. To reduce dissipation, we implemented here a directional limiter.
References


More References